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Multimedia Video Coding & Architectures (5LSE0), Module 03

Scalar Quantization

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Multimedia Video Coding and Architectures (5LSE0), Module 03

Mod 03, Preliminaries

Brief introduction: where are we...?

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System Overview

Analog capturing device (camera, microphone) → Sampling → Fine Quantization → PCM encoded or "raw" signal ("wav", "bmp", ...)

PCM encoded or "raw" signal → Transform → Quantizer → VLC encoding → Compressed bit stream (mp3, jpg, ...)

Compressed bit stream → Encipher → Error protect → Channel bit stream

Scalar Q (points to Quantizer)

COMPRESSION/SOURCE CODING (encompasses Transform, Quantizer, VLC encoding)

CHANNEL CODING (encompasses Encipher, Error protect)

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Why can Signals be Compressed? – (1)

Because signal amplitudes are statistically redundant

quantizer representation levels

Question 1:
What is the *shortest* average codeword length that one can achieve for a given signal (or "source")?

$$H(S) = -\sum_{i=1}^N P_S(s_i) \log_2[P_S(s_i)] \quad (\text{bit})$$

$$H(X) = -\int_{-\infty}^{\infty} p_X(x) \log_2[p_X(x)] dx$$

Question 2:
How did you obtain those codewords?

Huffman Coding
Run-length coding

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Why can Signals be Compressed? – (2)

Because signal amplitudes are mutually dependent

Question 1:
What is the *best possible* exploitation of the correlation (dependencies) in natural signals?
(Rate-Distortion Theory)

Question 2:
How do we *implement* a system that exploits the correlation in natural signals?
*(Compression algorithms: - DPCM
- Subband/wavelet
- Transform/DCT
- Motion compensation)*

To be discussed later in course

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Why can Signals be Compressed? – (3)

Because infinite accuracy of signal amplitudes is (perceptually) irrelevant

Question 1:
What is the *best possible* trade-off between required bit rate and resulting distortion?
(Rate-Distortion Theory)

Question 2:
How do we *implement* a system that gives us that best possible trade-off?
(Scalar and Vector Quantization Theory)

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5LSE0 - Mod 03 Part 1 Characterization of Quantizers

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Quantization Process – (1)

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Quantization Process – (2)

- * **Quantization:** Map a continuous-valued signal value $x(n)$ onto a (limited set of) discrete-valued signals values $y(n)$:

$$y(n) = Q[x(n)]$$
 such that $y(n)$ is good approximation of $x(n)$.
- * **Important:** # bits to represent $y(n)$.
(= average codeword length per representation level)

Design/Optimization Problem

- * What positions of the “decision levels”
- * What positions of the “representation levels”

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Quantization Process – (3)

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Formal Quantization Definition

- * Given amplitude x , divide domain of x into K non-overlapping intervals S_k :

$$S_k = \{x \mid x_k < x \leq x_{k+1}\} \quad k = 1, 2, \dots, K$$
- * If x falls in S_j then it is represented by y_j .
- * Quantization gives coding errors or quantization noise

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Quantizer Characteristics – (1)

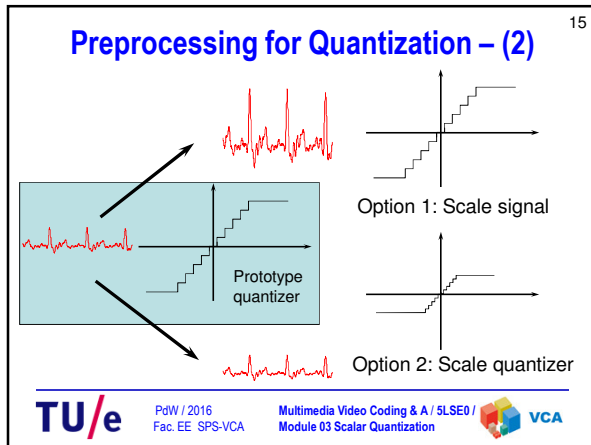
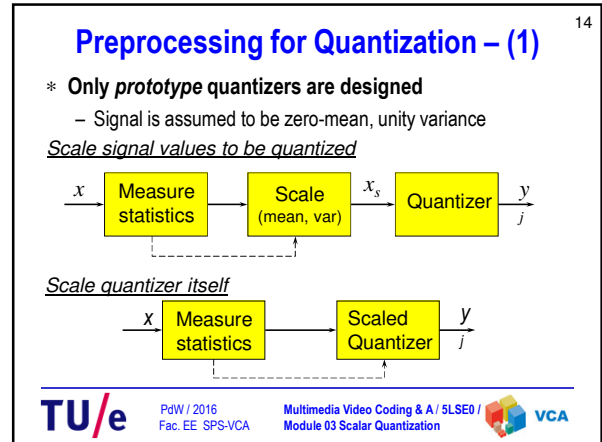
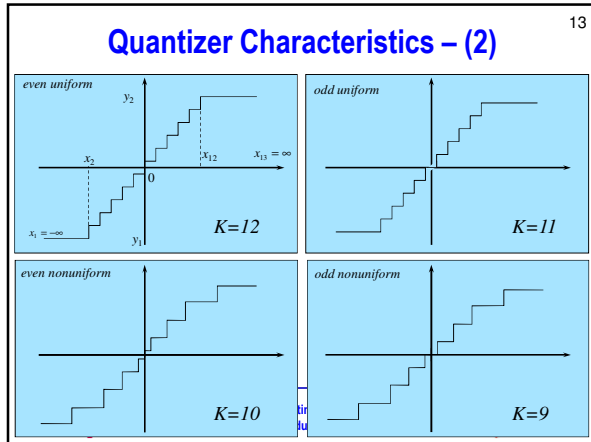
- * Signals values are assumed to lie symmetrically around zero

➡ Quantizers are usually *symmetric about origin*

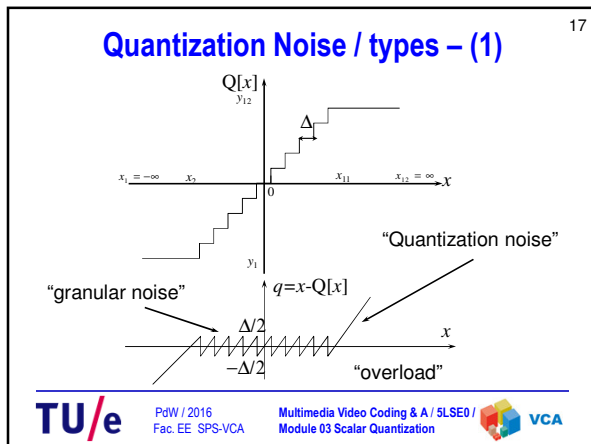
- * Quantizer choices:
 - Odd or even number of levels?
 - Uniform or non-uniform quantizers?
 - Way of optimizing the quantizer?

} “structure”
} “design”

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- ### Quantizer Characteristics – (3) 16
- * **Design of quantizer is optimization problem**
 – Find optimal values for decision and representation levels
- * **Two approaches:**
- Exploit knowledge of probability density function (PDF) of x to find optimum given #representation levels
 - Uniform quantization: easily solved and implemented
 - Non-uniform quantization: more difficult with limited gain
 - Select uniform quantizer with variable coarseness; determine effective #representation levels during applications
 - More practical; this is what JPEG/MPEG do
 - Choices inspired by theory
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Quantization Noise / Variance – (2) 18

* **Quantification of (average) quantization error:**
Variance of the quantization noise: $q = x - Q[x]$

* **Need to model probability density of x :** $p_X(x)$

* **Quantization noise variance:**

$$\sigma_q^2 = \int_{-\infty}^{\infty} \underbrace{(x - Q[x])^2}_{\text{Amount of error}} \underbrace{p_X(x)}_{\text{Probability of amount}} dx$$

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Quantization Noise / Variance – (3)

split into K - intervals

$$\sigma_q^2 = \int_{-\infty}^{\infty} (x - Q[x])^2 p_X(x) dx = \sum_{k=1}^K \int_{x_k}^{x_{k+1}} (x - y_k)^2 p_X(x) dx$$

Normalized measure: Signal-to-Noise-Ratio (SNR):

$$SNR = 10 \log_{10} \left(\frac{\sigma_x^2}{\sigma_q^2} \right) \text{ (dB)}$$

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Bit Rate versus Distortion – (1)

Graph 1: σ_q^2 vs σ_x^2 (32 levels)

Graph 2: σ_q^2 vs σ_x^2 (1 level)

Plot: σ_q^2 vs σ_x^2 (32 levels)

#representation levels
~ average code word length
~ bit rate

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Terminology for using quantizers

Encoder

x → Quantizer → y → VLC encoding → 00101101

- * Representation levels coded in $\lceil \log_2(K) \rceil$ bits:
Fixed-rate quantization
- * Representation levels coded in $H(Y)$ bits:
Quantization with entropy encoding
- * (Later: **Entropy-constrained quantization**)

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Bit Rate versus Distortion – (2)

- * Performance of quantizer is determined by
 - the number of representation levels (bit rate or average codeword length R)
 - the quality σ_q^2 or SNR
- * Fixed-rate quantizer design:
For given K , find the quantizer characteristic with smallest σ_q^2

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5 LSE0 - Mod 03, Part 2 Uniform Quantization

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R-D Optimal Design: Uniform Quantizer – (1)

* Given:

$$x_{k+1} - x_k = \Delta \quad y_k = \frac{(x_{k+1} + x_k)}{2}$$

* Find Δ such that σ_q^2 is minimized

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R-D Optimal Design: Uniform Quantizer – (2)

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$$\min_{\Delta} \sum_{k=1}^K \int_{x_k}^{x_{k+1}} (x - y_k)^2 p_X(x) dx$$

For values of $K > 3$, this requires numerical optimization

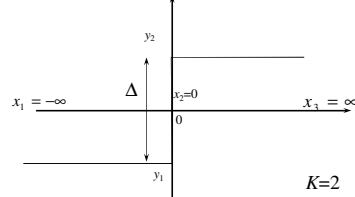
R bit/sample	$\Delta/\sigma_x = 1$			SNR (dB)		
	Uniform	Gaussian	Laplace	Uniform	Gaussian	Laplace
1	1.732	1.596	1.414	6.02	4.40	3.01
2	0.866	0.996	1.087	12.04	9.25	7.07
3	0.433	0.586	0.731	18.06	14.27	11.44
4	0.217	0.335	0.461	24.08	19.38	15.96
5	0.108	0.188	0.280	30.10	24.57	20.60
6	0.054	0.104	0.166	36.12	29.83	25.36
7	0.027	0.057	0.096	42.14	35.13	30.23
8	0.013	0.031	0.055	48.17	40.34	35.14

smaller Δ : "finer" quantizer → more difficult to quantize

Example: Two-Level Quantizer – (1)

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* Find optimal value of Δ for 2 level quantizer



$$\sigma_q^2 = \sum_{k=1}^K \int_{x_k}^{x_{k+1}} (x - y_k)^2 p_X(x) dx = \int_{-\infty}^0 (x + \frac{\Delta}{2})^2 p_X(x) dx + \int_0^{\infty} (x - \frac{\Delta}{2})^2 p_X(x) dx$$

Two-Level Quantizer – (2)

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$$\sigma_q^2 = \int_{-\infty}^0 (x + \frac{\Delta}{2})^2 p_X(x) dx + \int_0^{\infty} (x - \frac{\Delta}{2})^2 p_X(x) dx$$

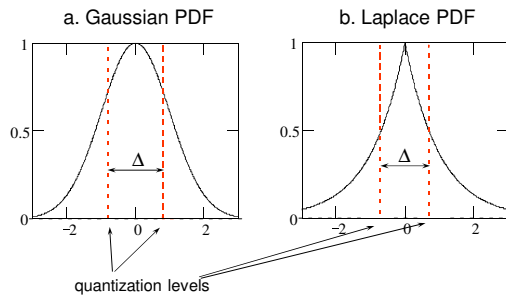
$$= \sigma_x^2 + \frac{\Delta^2}{4} - 2\Delta \int_0^{\infty} x p_X(x) dx$$

$$\Rightarrow \Delta_{\text{optimal}} = \min_{\Delta} \sigma_q^2 = 4 \int_0^{\infty} x p_X(x) dx = 2E[|X|]$$

PDF	Δ/σ_x	σ_q^2	SNR (dB)
Uniform	1.732	0.250	6.02
Gaussian	1.596	0.363	4.40
Laplace	1.414	0.500	3.01
Gamma	1.154	0.667	1.76

Two-Level Quantizer – (3)

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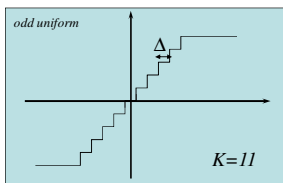


Implementation of Uniform Quantizer

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* The quantizer

- Odd
- Uniform
- $K = 11$ outp. levels



can be implemented by:

$$Q[x] = y_j = \Delta \text{nint}\left(\frac{x}{\Delta}\right) = \Delta \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor$$

$$j = \text{nint}\left(\frac{x}{\Delta}\right) = \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor$$

5LSE0 Mod 03, Part 3 Non-Uniform Quantization

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Non-Uniform Quantizer / 2 Approaches 31

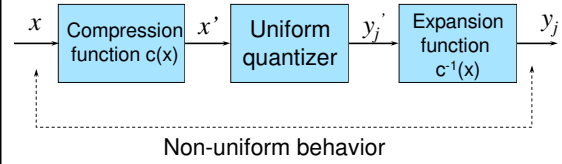
1. Preprocessing of x by non-linear function followed by uniform quantizer:

- Companding (compression-expanding)
- Simple implementation
- Popular for audio: logarithmic curves
 - A-law (Europe)
 - and μ -law (USA, Japan)

2. Lloyd-Max quantizers, minimization of σ_q^2

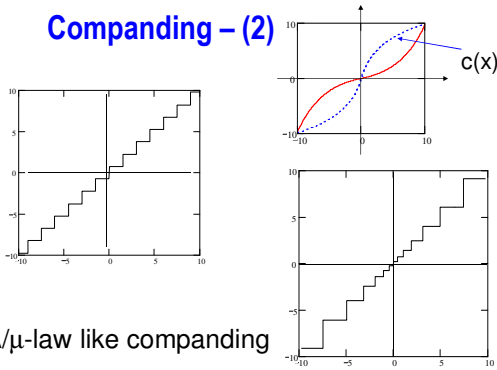
- Complex design
- More complex implementation than uniform quantizer
- Additional gain?

Companding – (1) 32



Uniform quantizer is embedded between two non-uniform functions

Companding – (2) 33



A/ μ -law like companding

A- and μ -Law Definition 34

$$\text{A-Law } c(x) = \begin{cases} x_{\max} \frac{ax/x_{\max}}{1 + \ln(a)} & 0 \leq x/x_{\max} \leq 1/a \\ x_{\max} \frac{\ln(ax/x_{\max})}{1 + \ln(a)} \text{sgn}(x) & 1/a < x/x_{\max} \leq 1 \end{cases}$$

μ -Law

$$c(x) = x_{\max} \frac{\ln(1 + \mu x/x_{\max})}{\ln(1 + \mu)} \text{sgn}(x)$$

Lloyd-Max Quantizer – (1) 35

- * Minimizes quantization noise variance, without enforcing any structure onto decision thresholds and representation levels

$$\min \sigma_q^2 = \min \sum_{k=1}^K \int_{x_k}^{x_{k+1}} (x - y_k)^2 p_X(x) dx$$

for:

$$x_k \quad k = 2, 3, \dots, K$$

$$y_k \quad k = 1, 2, \dots, K$$

* except for symmetry of the quantizer

Lloyd-Max Quantizer – (2) 36

- * General solution is given by implicit expressions:

$$x_k = \frac{1}{2}(y_{k-1} + y_k) \quad y_k = \frac{\int_{x_k}^{x_{k+1}} x p_X(x) dx}{\int_{x_k}^{x_{k+1}} p_X(x) dx}$$

- Note the structure of the formulas
 - Middle of two representation levels
 - Weighted average decision levels

Lloyd-Max Quantizer – (3)

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*** Property of solution:**
 - if x_k are known $\Rightarrow y_k$ are known
 - if y_k are known $\Rightarrow x_k$ are known

*** Iterative design necessary** initial choice, "seed"
 $x_1 = -\infty$

Assume: $x_2 = g \Rightarrow y_1 = \frac{\int_{-\infty}^g xp(x)dx}{\int_{-\infty}^g p(x)dx} \Rightarrow y_2 = 2x_2 - y_1$

Solve x_3 from: $y_2 = \frac{\int_{-\infty}^{x_3} xp(x)dx}{\int_{-\infty}^{x_3} p(x)dx} \Rightarrow y_3 = 2x_3 - y_2$

etcetera

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Lloyd-Max / Iterative Solution Scheme

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Lloyd-Max / Example iterative Q design

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Iteration				
	1	2	3	4
Assume x_2	-1.000	-0.500	-0.75	-0.612
y_1	-1.525	-1.141	-1.329	-1.224
y_2	-0.475	0.141	-0.171	0.000
y_3	-0.028	0.827	0.368	1.224
x_3	0.419	1.513	0.909	0.612

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Examples of Lloyd-Max Quantizers

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• Example from VcDemo Quantizer
*** For uniform PDF, Lloyd-Max quantizer is uniform**

PDF	k	K=2, 1 bit/sample		K=4, 2 bit/sample		K=8, 3 bit/sample		K=16, 4 bit/sample	
		x_k	y_k	x_k	y_k	x_k	y_k	x_k	y_k
Gaussian	1	0.000	0.798	0.000	0.453	0.000	0.245	0.000	0.128
	2			0.982	1.510	0.501	0.756	0.258	0.388
	3					1.050	1.344	0.522	0.657
	4					1.748	2.152	0.800	0.942
	5							1.099	1.256
	6							1.437	1.618
	7							1.844	2.069
	8							2.401	2.733
Laplace	1	0.000	0.707	0.000	0.402	0.000	0.233	0.000	0.124
	2			1.127	1.834	0.533	0.833	0.264	0.405
	3					1.253	1.673	0.567	0.729
	4					2.380	3.087	0.920	1.111
	5							1.345	1.578
	6							1.878	2.178
	7							2.597	3.017
	8							3.725	4.432

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Uniform quantizer versus Lloyd-Max

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For relevant bit rates, Lloyd-Max does often not pay off.

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Quantization with Entropy Coding – (1)

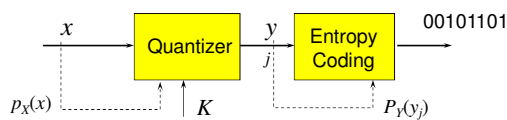
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Quantizer level more probable

• Example from VcDemo Quantizer

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Quantization with Entropy Coding – (2) 43



1. Quantizer is optimized for given #representation levels K
2. Entropy coder (VLC) is optimized for given probabilities of y_j , yielding average codeword length L

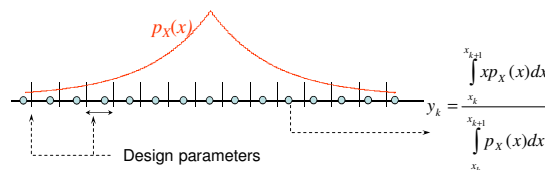
But, the combination is *not* optimal

- other quantizer / VLC combinations exist that gives a smaller quantization error variance for same average codeword length

Entropy-Constrained Quantization 44

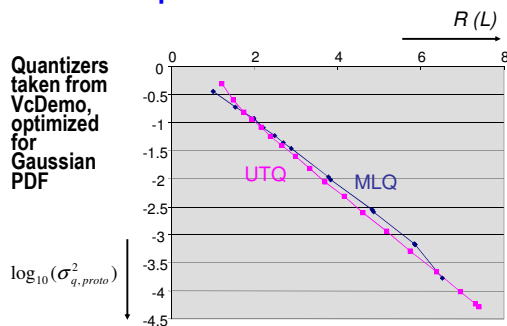
* To find the overall optimal result, quantizer and entropy coder must be *jointly* designed

- Complex optimization problem
- Reasonable approximations are obtained by Uniform Threshold Quantizers (UTQ)



Example MLQ versus UTQ 45

* Quantizers taken from VcDemo, optimized for Gaussian PDF

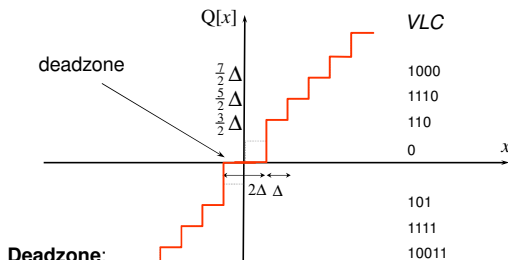


5LSE0 – Mod 03, Part 4 Quantization in practical cases 46

Quantization in Practice – (1) 47

- * Uniform quantizers are preferred
 - Implementation and limited loss compared to Lloyd-Max
 - Easily scalable (one parameter: step size Δ)
- * **Odd** quantizers are often preferred *over even* because of the presence of a representation level at zero
 - In good compression scheme many (near-)zero values occur
 - Zeroes efficiently coded by an entropy coder (run length coding)
- * **Audio**: Companding is usual
- * **Image/video coding**: No companding
 - Uniform quantizer with *deadzone* is typical

Quantization in Practice – (2) 48



Deadzone:

- Improves noise robustness of coding system
- “Stimulates” truncation to zero: can be coded efficiently

Overall Quantization Error – (1) 49

* Only *prototype* quantizers are designed with σ_q^2

Sending side

Receiving side

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Overall Quantization Error – (2) 50

* **Scaling procedure:** $x_s = \frac{x - \mu_x}{\sigma_x}$

* **Rescaling procedure:** $\hat{x} = \sigma_x y + \mu_x = \sigma_x \hat{x}_s + \mu_x$

* **Overall effect: Quantization noise variance is scaled with:** σ_x^2

$$\sigma_q^2 = E[(\hat{x} - x)^2] = E[(\sigma_x \hat{x}_s - \sigma_x x_s)^2] = \sigma_x^2 E[(\hat{x}_s - x_s)^2]$$

$$= \sigma_x^2 \sigma_{q, \text{prototype}}^2$$

⇒ **Quantization noise variance is linearly proportional to the variance of the signal to be quantized**

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Quantizer Example - (2) 52

[Back to ML Quantizers](#)
[Back to Prob. Calculation](#)

	Repres. Level	Probability	Codewords (positive)	Codewords (negative)
1	±9.247	0.00007	0001010101010	0001010101011
2	±6.875	0.00031	00010101011	000101010100
3	±5.519	0.00077	000101011	00010101100
4	±4.562	0.00152	000101000	0001010001
5	±3.823	0.00269	000101001	000101010
6	±3.217	0.00444	00011010	00011011
7	±2.703	0.00699	1011010	1011011
8	±2.253	0.01068	0001011	0001100
9	±1.855	0.01590	000111	101100
10	±1.497	0.02318	10111	000100
11	±1.175	0.03316	01000	01001
12	±0.884	0.04658	1100	1101
13	±0.622	0.06441	0101	1010
14	±0.388	0.08797	111	0000
15	±0.180	0.11987	011	100
16	0.000	0.16292	001	

* Simple binary coding would require $\lceil \log_2(31) \rceil = 5$ bits

* $H(S) = 3.876$ bit/repr.level

* $L = 3.912$ bit/repr.level

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Application to Signal Coding 53

[Back to Quantizers](#)

* We will see that efficient “**transforms**” used in compression produce

- A lot of “zero” values
- Some (significant) non-zero values

* **Typical symbol sequences to be coded:**
“5 1 0 0 0 0 0 0 3 0 0 6 0 0 0 1 0 0 0 0 ...”

- Will be done by {zero-run, non-zero symbol} pairs
- Here: “{0,5}, {0,1}, {7,3}, {2,6}, {4,1}, ...”
- The pairs will now be assigned a Huffman code

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Why can Signals be Compressed? 54

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Because signal amplitudes are mutually dependent

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The following sheets are included for further reference

Note on Quantization Noise Variance

- * We wish to calculate the variance of q :

$$\sigma_q^2 = E[q^2] - \underbrace{E[q]^2}_{=0} = \int_{-\infty}^{\infty} q^2 p_q(q) dq$$

- * Try to calculate $p_q(q)$, but with non-linear relation of $q=x-Q[x]$, this is not easy

- * Much easier, take following short-cut (E-operator):

$$\begin{aligned} \sigma_q^2 &= E[q^2] = E[(x - Q[x])^2] \\ &= \int_{-\infty}^{\infty} (x - Q[x])^2 p_x(x) dx \end{aligned}$$

Modeling of Quantizers

- * Quantizer is often the only non-linear element in a compression system
- * Linear (approximating) models are sometimes used to analyze the performance of a quantizer / compression system

