

Multimedia Video Coding & Architectures (5LSE0), Module 05

Differential PCM (DPCM) and Linear Predictive Coding (LPC)

Peter H.N. de With

(p.h.n.de.with@tue.nl)

slides version 1.0



PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0 /
Module 05 DPCM & Linear Pred. Coding



5LSE0 - Mod 05 Part 1 Correlation and Prediction



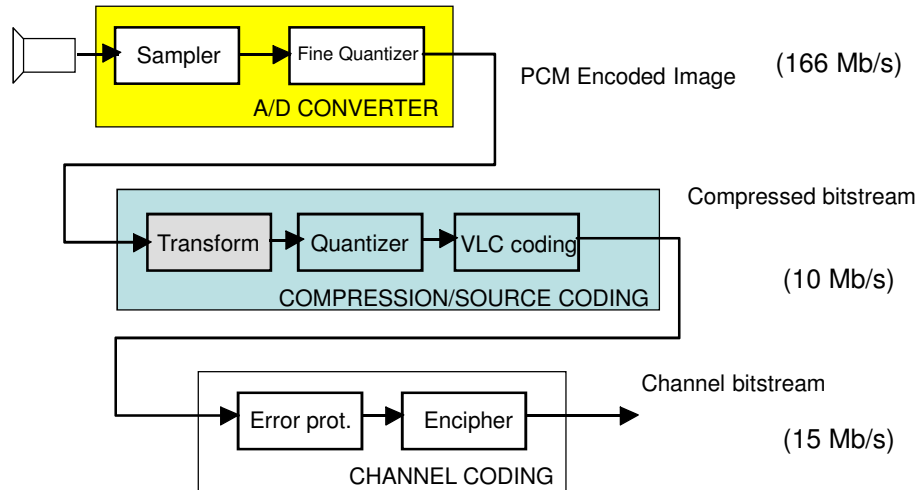
PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0 /
Module 05 DPCM & Linear Pred. Coding



General Compression System

3



TU/e

PdW / 2016
Fac. EE SPS-VCA

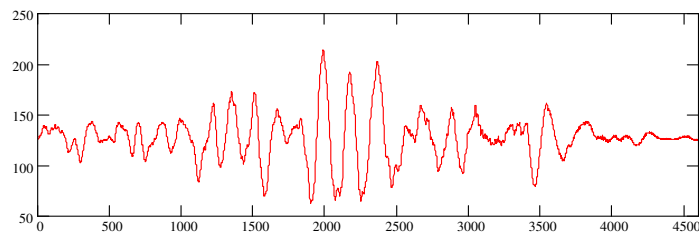
Multimedia Video Coding & A / 5LSE0 /
Module 05 DPCM & Linear Pred. Coding



Correlation in Signals – (1)

4

- * **Meaningful signals are often highly predictable:**
From the behavior in the past, the future signal values can be approximately estimated



- * **(Linear) Predictability has something to do with the autocorrelation function, so ...**

TU/e

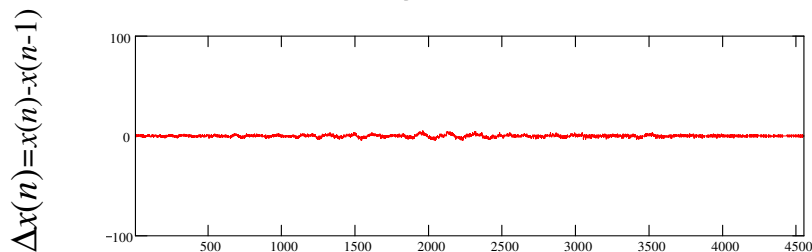
PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0 /
Module 05 DPCM & Linear Pred. Coding



Correlation in Signals – (2)

- * Differential PCM works on the principle that:
Anything that can be predicted from the signal's past can be reconstructed by the decoder
- * Simple example: Predict next signal value on basis of actual one. Send only differences to the decoder



Effect of Prediction on Quantization

- * If the same quantizer is used on $x(n)$ and $\Delta x(n)$, then compare:

$$\text{For } x(n) \quad \sigma_q^2 = \sigma_x^2 \sigma_{q,\text{prototype}}^2$$

$$\text{For } \Delta x(n) \quad \sigma_q^2 \neq \sigma_{\Delta x}^2 \sigma_{q,\text{prototype}}^2$$

Quality *scales with variance* of “signal to be quantized”

Principle of Differential PCM

* **DPCM encoder** consists of three steps:

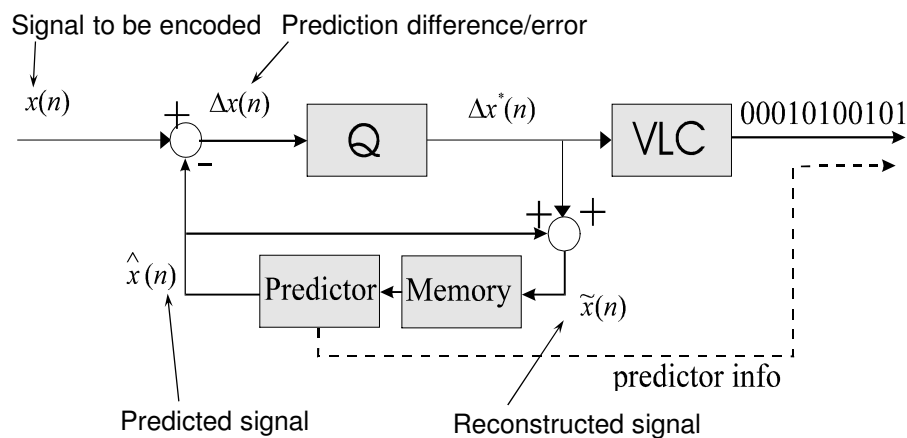
- **Predict** the current signal value $x(n)$ from past values $x(n-1), x(n-2), \dots$
- **Quantize** the difference signal = prediction error
- **Encode**, e.g. VLC the prediction difference (prediction error)

* **DPCM Decoder** reproduces same prediction of $x(n)$

* **Prediction in encoder** is based on quantized values

$$\tilde{x}(n-1), \tilde{x}(n-2), \dots$$

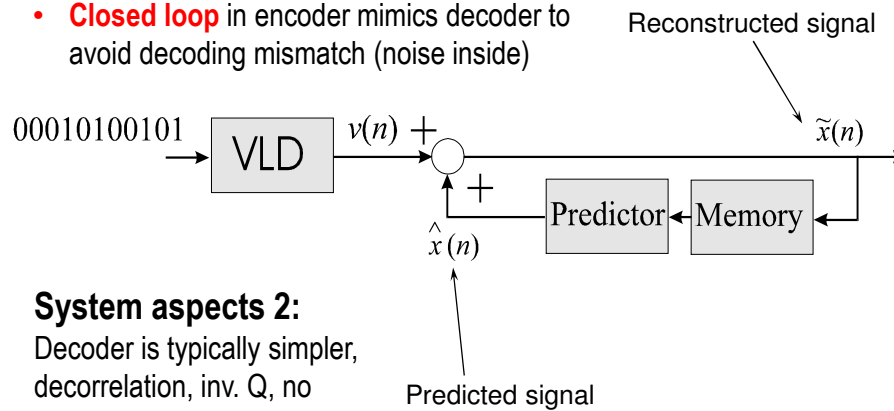
Diagram of DPCM Encoder & Signals



DPCM Decoder Diagram

System aspects 1: Loop

- **Closed loop** in encoder mimics decoder to avoid decoding mismatch (noise inside)



System aspects 2:

Decoder is typically simpler, decorrelation, inv. Q, no decision making

What Linear Predictor to Use?

* What is “best” predictor?

- Theoretically: $\hat{x}(n) = E[x(n)|\tilde{x}(n-1), \tilde{x}(n-2), \dots, \tilde{x}(n-N)]$
- Practically: Linear predictors only

* Examples:

- PCM $\hat{x}(n) = 0$
- Simple differences $\hat{x}(n) = \tilde{x}(n-1)$
- Average last two samples $\hat{x}(n) = h_1 \tilde{x}(n-1) + h_2 \tilde{x}(n-2)$
- General linear predictor $\hat{x}(n) = \sum_{k=1}^N h_k \tilde{x}(n-k)$

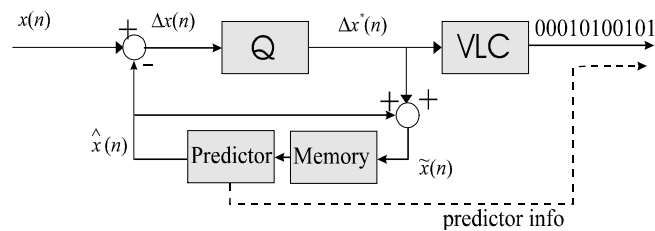
* How to evaluate “success/usefulness” of prediction (compared to PCM): *Prediction Gain*

5LSE0 - Mod 05

Part 2

Reconstruction Error, Prediction Gain and Optimal Prediction

Reconstruction Error in DPCM – (1)



* **Reconstruction / Coding error:**

$$\begin{aligned}
 r(n) &= x(n) - \tilde{x}(n) \\
 &= x(n) - [\hat{x}(n) + \Delta x^*(n)] \\
 &= \Delta x(n) - \Delta x^*(n) \\
 &= q(n)
 \end{aligned}$$

Reconstruction Error in DPCM – (2)

- * Overall **reconstruction** error in DPCM is equal to the **quantization** error of $Q[\Delta x(n)]$
- * DPCM reconstruction error variance: $\sigma_r^2 = \sigma_q^2$
- * SNR will be determined by quantizer performance, relative to the **variance of the input signal** $\Delta x(n)$

Rate-Distortion Model

- * **Model for relation bit-rate vs. distortion:**
 - Gaussian independent data samples
 - Variance σ_x^2
$$R(D) = \frac{1}{2} \log_2 \frac{\sigma_x^2}{D} \quad (\text{bits/sample})$$
- * **For PCM:**

$$R_{PCM} = \frac{1}{2} \log_2 \frac{\sigma_x^2}{\sigma_q^2}$$
- * **For DPCM:**

$$R_{DPCM} = \frac{1}{2} \log_2 \frac{\sigma_{\Delta x}^2}{\sigma_q^2}$$

(Coding) Gain Obtained by Prediction

- * Theoretical difference in bit rate if **same quantizer** (with same σ_q^2) is used (**same quality of encoded signal**):

$$R_{PCM} - R_{DPCM} = \frac{1}{2} \log_2 \frac{\sigma_x^2}{\sigma_{\Delta x}^2} \quad \leftarrow \text{Ratio of variances!}$$

- * Advantage of DPCM over PCM depends on the **ratio of signal variance: prediction error variance**:

– Prediction gain: $G_p = \frac{\sigma_x^2}{\sigma_{\Delta x}^2}$

Prediction Gain Example

Case	G_p	Comments
$\sigma_{\Delta x}^2 < \sigma_x^2$	$G_p > 1$	DPCM outperforms PCM
$\sigma_{\Delta x}^2 = \sigma_x^2$	$G_p = 1$	DPCM identical to PCM
$\sigma_{\Delta x}^2 > \sigma_x^2$	$G_p < 1$	DPCM worse than PCM
$\sigma_{\Delta x}^2 = 0$	$G_p \rightarrow \infty$	Perfect prediction

Sometimes the prediction gain is given as: $\frac{1}{2} \log_2 G_p$

- Practical example: $G_p = 20 \Rightarrow$ Gain bit rate: 2.1 bits/sample

The Optimal Predictor – (1)

- * **Maximal benefit of DPCM is obtained if G_p is large**
 - Make variance of prediction difference $\sigma_{\Delta x}^2$ as **small** as possible

- * **Rationale**

- Make the prediction $\tilde{x}(n)$ as close as possible to $x(n)$ in quadratic sense:

- Theoretical $\sigma_{\Delta x}^2 = E[(x(n) - \tilde{x}(n))^2]$

- Practical (est.) $\sigma_{\Delta x}^2 = \frac{1}{N} \sum_{\text{all } n} (x(n) - \tilde{x}(n))^2$

The Optimal Predictor – (2)

- * **Predictor:** $\hat{x}(n) = \sum_{k=1}^N h_k x(n-k)$
Find weights h_k

- * **Minimize:** $\sigma_{\Delta x}^2 = E[(x(n) - \hat{x}(n))^2]$

- note that the prediction takes place on original data

- * **Solution for N linear prediction coefficients h_k**

$$\frac{\partial \sigma_{\Delta x}^2}{\partial h_i} = 0 \quad \Rightarrow \quad R_X(i) = \sum_{k=1}^N h_k R_X(i-k) \quad i = 1, 2, \dots, N$$

(Normal or Yule-Walker Equations)

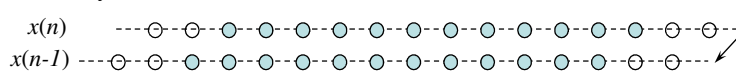
The Optimal Predictor – (3)

$$\begin{bmatrix} R_x(1) \\ R_x(2) \\ \vdots \\ R_x(N) \end{bmatrix} = \begin{bmatrix} R_x(0) & R_x(1) & \cdots & R_x(N-1) \\ R_x(1) & R_x(0) & & R_x(N-2) \\ \vdots & & \ddots & \vdots \\ R_x(N-1) & R_x(N-2) & \cdots & R_x(0) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{bmatrix}$$

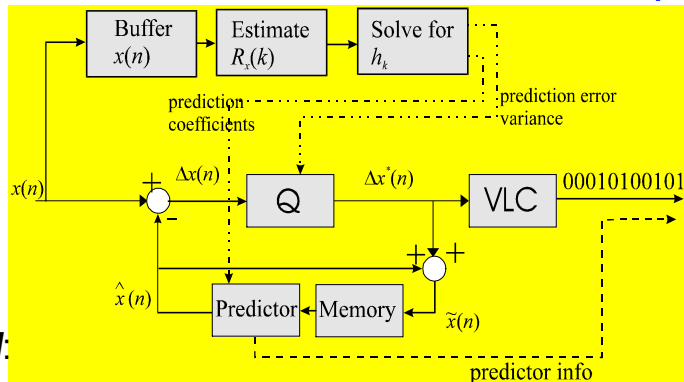
- * From this set of equations the N linear prediction coefficients can be computed.
- * Need to know $R_x(k)$: Autocorrelation function of $x(n)$

Estimation Autocorrelation Function – (1)²⁰

- * **Theoretical:** $R_X(k) = E[X(n)X(n-k)]$
- * **Practical choice:** $R_X(k) = \frac{1}{N} \sum_{n=0}^N X(n)X(n-k)$
- * **Practical problems:**
 - Only limited amount of data (especially in **adaptive** versions)
 - Boundary effects:


 - Different “estimators” for autocorrelation function exist, with theoretical and practical implications

Estimation Autocorrelation Function – (2) 21

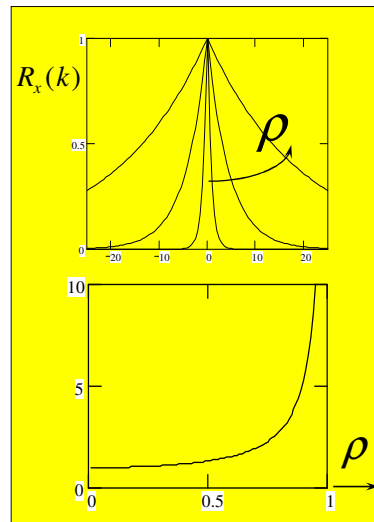


Overhead:

- Prediction coefficients h_k for the signal
- Variance of prediction difference: used to scale the quantizer
- Mean of signal (usually first subtracted)
- Start values for decoding process (boundary values)

Example of Prediction Gain (1-D) 22

- * **Model:** $R_x(k) = \sigma_x^2 \rho^{|k|}$
- * **Predictor:** $\hat{x}(n) = h_1 x(n-1)$
- * **“Set” of linear equations:** $R_x(1) = R_x(0)h_1$
- * **Solution:** $h_1 = \frac{R_x(1)}{R_x(0)} = \rho$
- * **Resulting prediction error variance:** $\sigma_{\Delta x}^2 = \sigma_x^2(1 - \rho^2)$
- * **Prediction gain:** $G_P = \frac{1}{1 - \rho^2}$



DPCM Artifacts – Coding noise

23

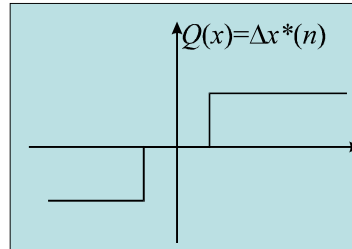
* **Granular noise (like PCM)**

* **Slope overload:**

- Prediction differences $\Delta x(n)$ too large for quantizer to handle
- Encoder cannot track rapidly changing signal values (i.e. at signal slopes)

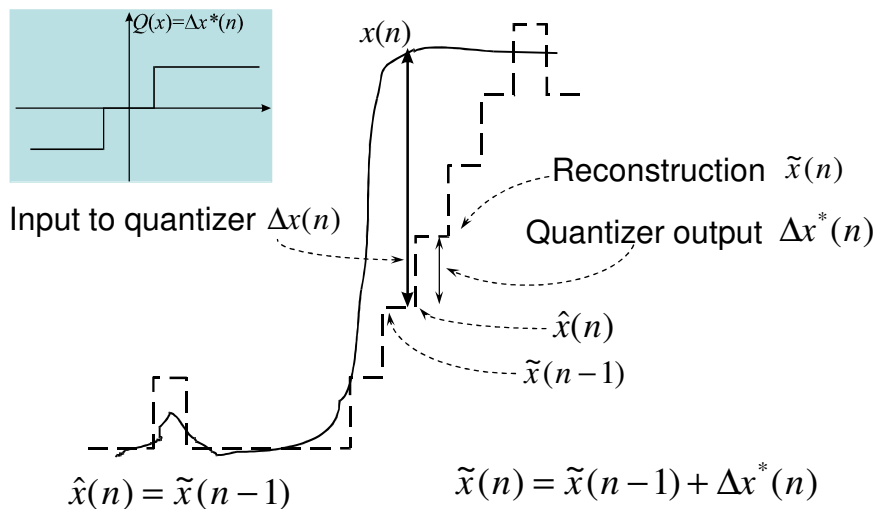
* **Example:**

- Encoder $\hat{x}(n) = \tilde{x}(n-1)$
- Decoder $\tilde{x}(n) = \hat{x}(n) + \Delta x^*(n)$
 $= \tilde{x}(n-1) + \Delta x^*(n)$



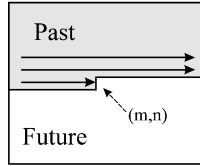
DPCM Slope Overload

24

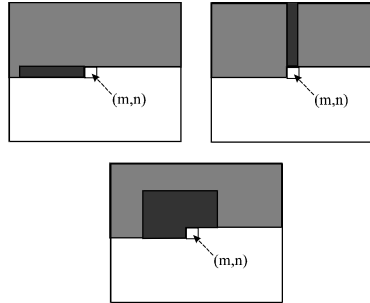


DPCM on Images / Video

- * Same principle as 1-D
- * Definition of “Past” and “Future” in images:



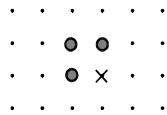
- * Predictions:
 - horizontal (scan line)
 - vertical (column)
 - two-dimensional



Linear Predictor for Images

- * Typical linear predictor for images:

$$x(m, n) = h_{01}x(m, n - 1) + h_{10}x(m - 1, n) + h_{11}x(m - 1, n - 1)$$

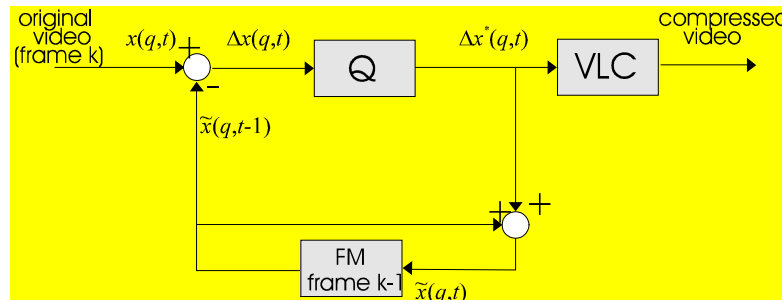


- * Yule-Walker for 2-D predictor: Solve for optimal prediction coefficients

$$\begin{bmatrix} R(0,1) \\ R(1,0) \\ R(1,1) \end{bmatrix} = \begin{bmatrix} R(0,0) & R(1,-1) & R(1,0) \\ R(1,-1) & R(0,0) & R(0,1) \\ R(1,0) & R(0,1) & R(0,0) \end{bmatrix} \begin{bmatrix} h_{01} \\ h_{10} \\ h_{11} \end{bmatrix}$$

Need 2-D correlation function!

DPCM on Video pictures

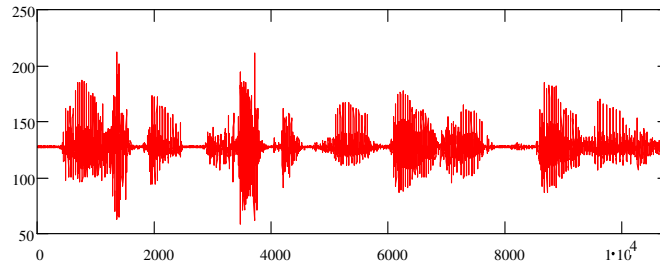


- * Usually difference between subblocks in the frames (e.g. 16x16 pixels)
- * Often fixed simple predictor: $\hat{x}(q, t) = \tilde{x}(q, t - 1)$
 - No need to estimate prediction coefficients

5LSE0 - Mod 05 Part 3 Adaptive Prediction and Systems

Adaptive DPCM systems

29



- * **Apply the same predictor everywhere is not a good idea, because signal correlation changes over time**
 - Adapt predictor (and quantizer) locally
 - Need to estimate autocorrelation function locally
 - Incurs overhead

TU/e

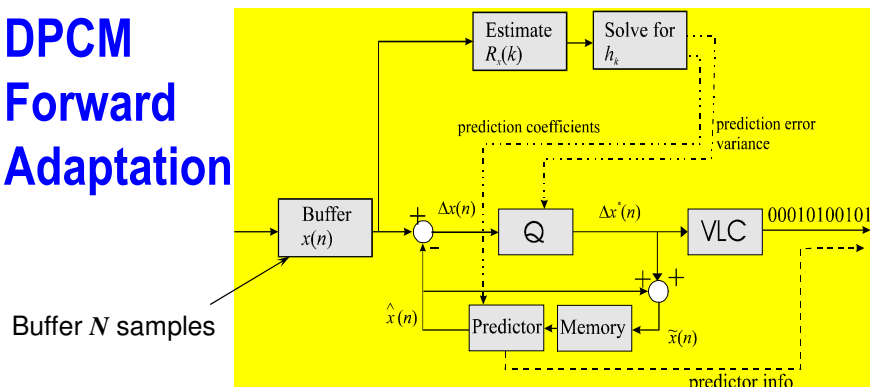
PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0 /
Module 05 DPCM & Linear Pred. Coding



DPCM Forward Adaptation

30



- Effective adaptation needs small segments (e.g. speech: 32 msec, video?)
- Small segments:
 - generate a lot of overhead (prediction coefficients)
 - difficult to estimate autocorrelation reliably
- Make trade-off on basis of prediction gain

TU/e

PdW / 2016
Fac. EE SPS-VCA

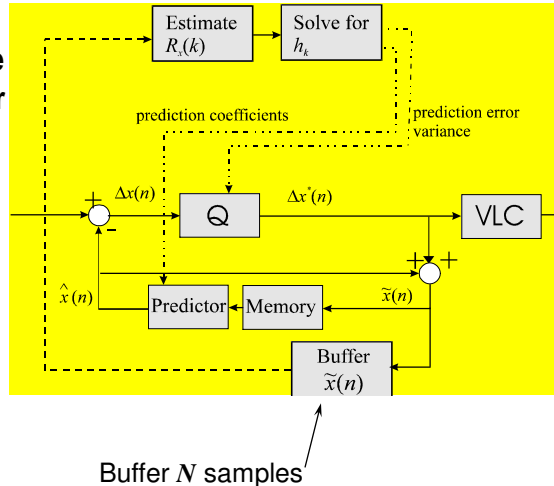
Multimedia Video Coding & A / 5LSE0 /
Module 05 DPCM & Linear Pred. Coding



DPCM with Backward Adaptation

Use information *previous* data segment to calculate prediction coefficients for *current* segment

- Decoder can reproduce the calculation of the prediction coefficients
- *Drawback:* Adaptation may come too late



Adaptation by Mode Selection

- * **In images, simplify the adaptation by**
 - only allowing to choose among few predictors
 - limiting the incurred overhead (e.g. per 8x8 block)
- * **Per block, the “best” predictor is selected**
 - “Best” means: largest prediction gain
- * **Example: Three predictors among which to choose**
 - $\tilde{x}(m, n) = \hat{x}(m - 1, n)$ “horizontal edge”
 - $\tilde{x}(m, n) = \hat{x}(m, n - 1)$ “vertical edge”
 - $\tilde{x}(m, n) = 0.4\hat{x}(m - 1, n) + 0.4\hat{x}(m, n - 1) + 0.2\hat{x}(m - 1, n - 1)$ no directional preference

Application: Speech Production and LPC

33

- * **Human speech is produced in the vocal tract as combination of vocal cords (in glottis) interacting with articulators in vocal tract**
 - Model this as a signal processing process
- * **Human speech perception exhibit masking**
 - Model this as a signal processing (masking) filter
- * **Results in DPCM, like speech coding:**
 - LPC Vocoder
 - Linear Predictive Coding (LPC)
 - Analysis-by-synthesis coding (CELP)

TU/e

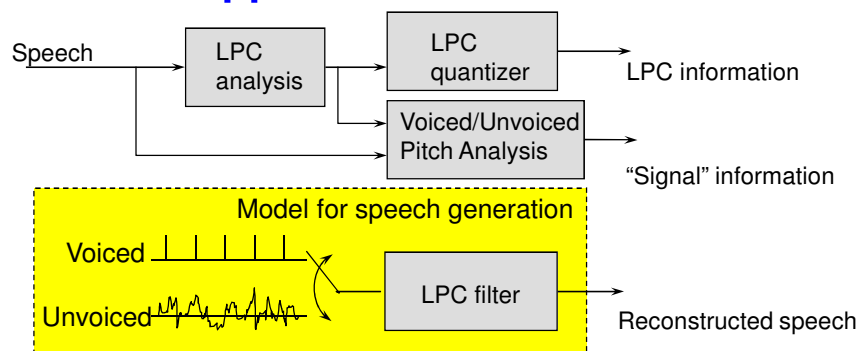
PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0/
Module 05 DPCM & Linear Pred. Coding



Application / LPC Vocoder

34



- * **Voice Coder (VoCoder) analyzes:**
 - Spectral properties of speech segment
 - Voice/Unvoiced segment information
 - Pitch of signal
- * **Objective: Understandable speech**

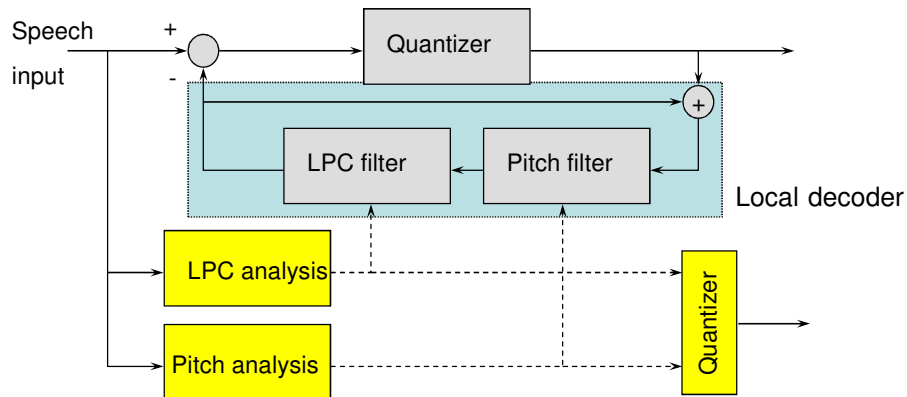
TU/e

PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0/
Module 05 DPCM & Linear Pred. Coding



Application / Adaptive LPC (APC Coder) 35



* Very similar to (adaptive) DPCM

* Objective: **Waveform precise reproduction**

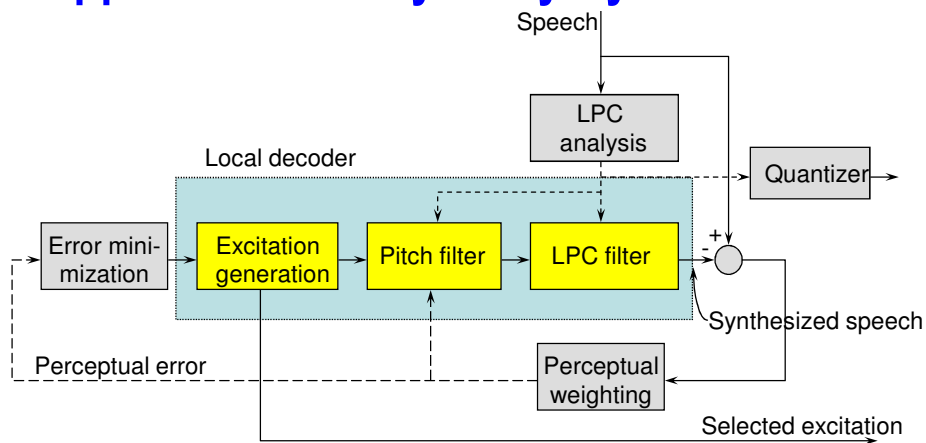
TU/e

PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0/
Module 05 DPCM & Linear Pred. Coding



Application / Analysis-by-Synthesis LPC 36



* Marriage between vocoder and APC/DPCM

* **CELP**: Codebook-excited linear prediction

TU/e

PdW / 2016
Fac. EE SPS-VCA

Multimedia Video Coding & A / 5LSE0/
Module 05 DPCM & Linear Pred. Coding



Standard Speech Coders using DPCM/LPC ³⁷

Standard number	ITU G.711	ITU G.726	ITU G.728	ETSI	ITU G.723.1
Year	1972	1990	1992/1994	1994	1995
Type of Coder	Companded PCM	ADPCM	LD-CELP	VSELP	ACELP ++
Bit rate	64 kbs	16-40 kbs	16 kbs	5.6 kbs	5.3 & 6.3 kbs
Quality	Toll	< Toll	Toll	GSM	< Toll
Delay	0.125 msec	0.125 msec	0.625 msec	20 msec	30 msec

Concluding statements ³⁸

- * **Original DPCM system encoder is a feedback loop**
 - Decoder reconstructs the same signal without side information
- * **Adaptive prediction and/or quantization improves SNR**
 - Adaptive prediction to image structure
 - Adaptive quantization for reducing noise in specific areas
- * **Model-based prediction such as synthesis enforce decorrelation, but increase the complexity of that stage**
- * **Buffering approach: trade-off complexity & performance**
 - Forward: accurate but expensive overhead and backward v.v.
- * **Predictive systems often embedded into larger systems**