Enabling Technologies for Sports (5XSF0), Module 03 Appendix

Appendix on Filter Design

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Overview Module 03 Filters Appendix

* **Objective: simple exercises in filter design**
  - Hands on experience of simple FIR filters
  - Computing of spectrum and filter behavior
* **Recapitulation**
  - Module 02: section on FIR filter architecture and time-domain equations
  - Module 03: DFT definition and frequency-domain filtering

3

Four representations of FIR filters

1. Difference equation

differentievergelijking

$y[n] = 1.5y[n-1] - 0.85y[n-2] + x[n]$

2. Impulse response

impuls response

$(h[n]):\ 1, 1.5, 1.4, 0.825, \ldots$

3.Frequency response

frequentieresponse

$$H(\Omega) = \frac{1}{1 - 1.5e^{-j\Omega} + 0.85e^{-j2\Omega}}$$

4.Transfer function

overdrachtsfunctie

$$H(z) = \frac{z^2}{z^2 - 1.5z + 0.85}$$

4

Non-recursive filters (FIR)

* **Output signal $y(n)$ depends only on input signal $x(n)$**

Niet-recursieve filters

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M]$$

Uitgangssignaal hangt alleen af van ingangssignaal

Niet-recursieve vergelijking voor impulsresponse

$h[n] = b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] + \ldots + b_M\delta[n-M]$

$h[0] = b_0$

$h[1] = b_1$

$h[2] = b_2$

$\ldots$

$h[M] = b_M$

Impulse response is within limited time interval!

b_k zijn opeenvolgende termen $h[k]$: FIR Finite Impulse Response

5

Different characterizations of FIR

* **Representations of FIR filter**
  - 1. Difference Equation $$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
  - 2. Frequency Response $$H(\Omega) = \sum_{k=0}^{M} b_k e^{-jk\Omega}$$
  - 3. Transfer function $$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$
* **Comments**
  - Find suitable behavior with low amount of coefficients
  - FIR filters are slower than recursive Inf. Impulse Response filters
  - FIR filters have transversal structure (no loops) thus always stable in behavior!

6

Example 1 FIR filter / Questions – (1)

* **Filter coefficients are { b_k } = {3, -1, 2, 1}**
  - 1. Compute response $y(n)$ for signal $x(n)$ = 2, 4, 6, 4, 2
  - 2. Transfer function
  - 3. Frequency response
  - 4. Impulse response

Example 1 FIR filter / Answers – (2)

7

1. $y[n] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$
   $(x[n]) : 2 \;\; 4 \;\; 6 \;\; 4 \;\; 2$ als invoer
   $\uparrow \; \uparrow \; \uparrow \; \uparrow \; \uparrow$
   $0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$
   $y[0] = 3x[0] = 6$
   $y[1] = 3x[1] - x[0] = 10$
   $y[2] = 3x[2] - x[1] + 2x[0] = 18$
   $y[3] = 3x[3] - x[2] + 2x[1] + x[0] = 16$ etc.
   $(y[n]) : 6 \;\; 10 \;\; 18 \;\; 16 \;\; 18 \;\; 12 \;\; 8 \;\; 2 \;\; 0$ als invoer
2. $H(z) = b_0 z^0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} = 3 - z^{-1} + 2z^{-2} + z^{-3} = z^{-3}(3z^3 - z^2 + 2z + 1)$
3. $H(\Omega) = 3 - e^{-j\Omega} + 2e^{-j2\Omega} + e^{-j3\Omega}$
4. $(h[n]) : 3 \;\; -1 \;\; 2 \;\; 1$

Example 2 FIR filter / Questions, Answers

8

Gegeven FIR filter met $\{b_k\} = \{1, 2, 1\}$

1. Bereken de amplitude en argument van frequentieresponse
2. Bereken het uitgangssignaal van $x[n] = 2e^{j\frac{\pi}{3}n}$

Antwoord

1. $H(\Omega) = 1 + 2e^{-j\Omega} + e^{-j2\Omega} = e^{-j\Omega}(e^{j\Omega} + 2 + e^{-j\Omega}) = e^{-j\Omega}(2 + 2\cos\Omega)$
   $|H(\Omega)| = 2 + 2\cos\Omega$
   $\arg\Big(H(\Omega)\Big) = -\Omega$
2. gain is $|H(\frac{\pi}{3})| = 2 + 2\cos\frac{\pi}{3} = 2 + 1 = 3$
   $\arg\Big(H(\frac{\pi}{3})\Big) = -\frac{\pi}{3}$
   $y[n] = 2 \cdot 3 \cdot e^{-j\frac{\pi}{3}} e^{j\frac{\pi}{3}n} = 6e^{j\frac{\pi}{3}(n-1)}$ faseverschuiving

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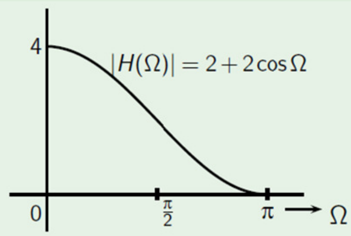
Example 2 FIR filter / Freq. response

Frequentierespons van FIR met $\{b_k\} = \{1, 2, 1\}$

* **Low-pass filter**

Laagdoorlaatfilter



Example 3 / 5-pnt Moving Average – (1)

5-punts moving-average filter (niet causaal)

Gegeven $y[n] = 0.2(x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2])$
Gevraagd:

1. impulsresponse
2. frequentieresponse
3. overdrachtsfunctie

Antwoord

1. $h[n] = 0.2(\delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2])$
2. $H(\Omega) = 0.2(e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}) =$
$0.2(1 + 2\cos\Omega + 2\cos 2\Omega)$
3. $H(z) = 0.2(z^2 + z + 1 + z^{-1} + z^{-2}) = \frac{0.2(1+z+z^2+z^3+z^4)}{z^2}$

Example 3 / 5-pnt Moving Average – (2)

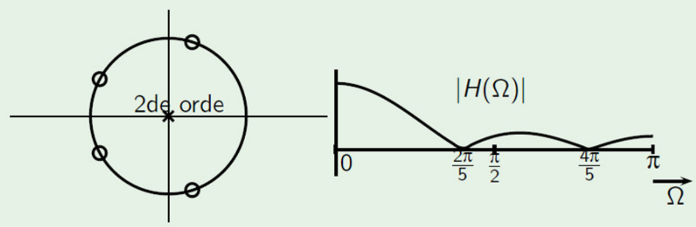
* **Poles and zero points of 5-point FIR**

$$H(z) = \frac{0.2(1+z+z^2+z^3+z^4)}{z^2} = \frac{0.2(1-z^5)}{z^2(1-z)}$$

$$1+z+z^2+z^3+z^4+z^5+\ldots = \frac{1}{1-z}$$

$$1+z+z^2+z^3+z^4 = \frac{1}{1-z} - z^5(1+z+z^2+z^3+\ldots) = \frac{1-z^5}{1-z} = 0$$

$z \neq 1$ en 4 nulpunten op eenheidscirkel



Example 3 MA FIR / Analysis – (3)

* **Causal or non-causal filter**
* 1. When FIR filter is symmetric about n=0, then FIR filter is non-causal
  – => Frequency response is real function in omega
  – => ***Zero-phase*** filter (no complex argument)
* 2. When FIR filter starts at n=0, then FIR filter is causal
  – => Frequency response is complex function in omega
  – => ***Linear-phase*** filter (argument is linear function in omega)